THEORY OF HIGH EFFICIENCY SECOND HARMONIC GENERATION OF HIGH INTENSE FEMTOSECOND PULSES

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The problem of high efficient second harmonic generation (quadratic non-linearity) of femtosecond laser pulses under the condition of laser radiation self-action (cubic non-linearity) is considered on bases of non-linear Shroedinger equation taking into account second order dispersion, phase and/or group mismatching and various life-times of cubic gratings. Various cases of generation are analyzed by both analytical and numerical methods. As a result methods and conditions for essential growth (up to 60%) SHG /generation are obtained.

SHG, high efficiency

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1. INTRODUCTION

Problem of high effective second harmonic generation (SHG) by femtosecond pulses remains unresolved for more than 15 years (Ref.1-6). At present there is no good technology to get high (more than 50%) conversion energy of SHG of high intense femtosecond (5-100fs) pulses in physical experiments. The main reason is the affection of self-action of laser pulse on the SHG process. This affection grows dramatically with the pulse intensity growth, causing dramatic reduce of conversion energy. It should be stressed that all the attempts to achieve high effective SHG of high intense femtosecond pulse on basis of phase and group matching failed and at present the efficiency of generation do not achieve 20% in physical experiments.

Various methods to increase SHG efficiency and their implementation are considered in the present paper far beyond the scope of traditional approach. Mathematical investigation is based on the non-linear Shroedinger equation with second order dispersion and without diffraction of light beam. In the framework of long pulse approximation the analitical solution was obtained due to the original approach using invariants of the process. The following cases of generation were analyzed: generation under phase and group velocity matching; generation under phase mismatching and group velocity matching; generation under phase and group velocity mismatching; generation far away from phase and group velocity matching; generation under the various life-times of cubic gratings. Bistable dependence of the solution in the region of strong self-action far from phase matching was obtained. Switching between the branches – spontaneous or induced – results in the essential growth of efficiency, and may be the main cause of high effective and high intensive regimes of generation at the pulse duration of certain values. Compensation of self-action and high level of SHG efficiency as a result can be achieved due to different values of lifetimes of cubic gratings. We show also the possibility of high effective generation under condition of big phase mismatching.

It should be noticed that using the obtained results one can create an optical processor with femtosecond time of switching and high ratio of power in the upper and low states.

2. BASIC EQUATIONS

The system of dimensionless equations that describe SHG process by femtosecond pulse taking into account self-action for the diffraction length greatly exceeding the total length of nonlinear medium or for the wave propagation in optical fibers is the following (Ref.2):

$$\begin{gathered}\frac{\partial A_1}{\partial z} + iD_1\frac{\partial^2 A_1}{\partial \eta^2} + i\gamma A_1^* A_2 e^{-i\Delta kz} \\ + i\alpha_1 A_1\left(\left|A_1\right|^2 + (1+\beta)\left|A_2\right|^2\right) = 0\end{gathered}, \quad 0 < z \le L_z$$

$$\begin{gathered}\frac{\partial A_2}{\partial z} + \nu\frac{\partial A_2}{\partial \eta} + iD_2\frac{\partial^2 A_2}{\partial \eta^2} + i\gamma A_1^2 e^{i\Delta kz} \\ + i\alpha_2 A_2\left((1+\beta)\left|A_1\right|^2 + \left|A_2\right|^2\right) = 0\end{gathered}, \qquad (1)$$

$$\alpha_2 = 2\alpha_1 = 2\alpha .$$

Here η - dimensionless time in the system of coordinates that accompanies the basic wave pulse, z - normalized longitudinal coordinate, $D_j \sim -0.5\frac{\partial\ ^2\bar{k}_j}{\partial\ \bar{\omega}_j^2}$ - coefficients that characterize second order dispersion, $\bar{k}_j$, $\bar{\omega}_j$ - scaled wave number and frequency of j wave correspondingly, γ - coefficient of nonlinear coupling of interaction waves, $\Delta k = k_2 - 2k_1$ - dimensionless mismatching of their wave numbers, α_j - coefficients of self-action of waves, A_j - complex amplitudes of harmonics ($j = 1,2$), normalized on the maximal amplitude of the first harmonic in the input section of the medium ($z = 0$). Parameter ν is proportional to the difference of the inverse values of group velocities of the second harmonic wave and the basic wave, L_z - length of nonlinear medium. Parameter β characterizes influence of various grating induced by interacting waves into self-action and its value varies from 0 to 1 (More general form of (1), taking into account difference of influence of difference gratings can be found in –Ref.4) .

At the input of nonlinear medium initial distribution of the basic wave pulse is determined

$$A_1(z=0,\eta) = A_1^{\,0}(\eta)\,,\ 0 \le \eta \le L_t \qquad (2)$$

in the form of gauss pulse

$$A_1^0\,(\eta\,) = A_{10}\exp\left(-((\ \ - L_t)/\ \ {}^2/2\right), \qquad (3)$$

where A_{10} - is dimensionless amplitude, L_t - dimensionless time. Amplitude of the second harmonic wave in the input section is equal zero: $A_2(z=0,\eta)=0$.

SHG process under conditions of self-acting of waves has the number of invariants (see for example paper –Ref.7), the values of the invariants should be controlled in the process of computer simulation.

Dimensionless amplitude A_{10} was equal 1. Conservative difference schemes that preserve difference analogs of system (1) invariants were used.

Efficiency of energy transformation from the basic wave to the second harmonic was evaluated traditionally as:

$$\theta(z) = \int_0^{L_t} \left|A_2(z,\eta)\right|^2 d\eta \Big/ \int_0^{L_t} \left|A_1(z=0,\eta)\right|^2 d\eta$$

3. LONG PULSE APPROXIMATION

It is possible to integrate system (1) in the framework of long pulse approximation ($\partial/\partial\eta = 0$) -Ref.7. Using the standard representation of complex amplitudes of basic and second harmonic waves

$$A_j = a_j e^{i\varphi_j}, \quad j = 1,2,$$

where amplitudes a_j and phases φ_j are real functions, we can get from (1) equation with regard to intensity $P_1 = a_1^2$

$$\int_{(P_1)_0}^{P_1} \frac{dx}{\sqrt{(1-x)\left[4\gamma^2 x^2 - (1-x)(2\alpha\beta x + \Delta k)^2\right]}} = \int_{(P_1)_0}^{P_1} \frac{dx}{\sqrt{f(x)}} = \mp z + C, \quad (4)$$

here $(P_1)_0$ - initial value of the basic wave energy.

Equation (5) was integrated explicitly using elliptical functions in –Ref.7, and regions on the plane of parameters ($q = \gamma^2 \big/ (\alpha\beta)^2$, $p = \Delta k \big/ 2\alpha\beta$), corresponding to various solutions, are shown (fig.1)

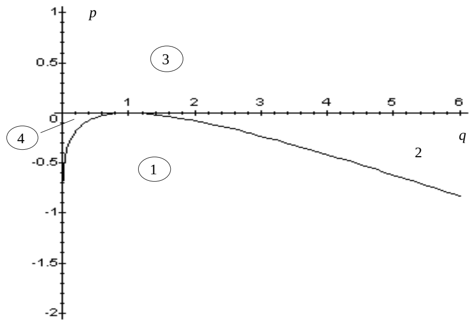


Fig.1. Regions of qualitatively different SHG regimes on the plane of parameters $q = \gamma^2 \big/ (\alpha\beta)^2$, $p = \Delta k \big/ 2\alpha\beta$

An important result follows from (4): parameter of self-action α appears in these expressions only being multiplied by β. Thus, in the framework of long pulses the case $\beta = 0$ is equivalent to the absence of self-action ($\alpha = 0$) and compensation of self-action by the appropriate chose of nonlinear medium can be achieved.

The most corresponding to the experiments with femtosecond pulses is the region of bistability 4, where two regimes of generation with the same characteristics (period, invariants) occur. One of them is high effective regime. The other that is realized in practice is low effective regime. Insertion of the number of simultaneous phase shifts for both waves at the appropriate sections of the medium can, in principle, switch the system from the low to the high effective branch of generation. We'll show later that insertion of even one phase shift can significantly increase efficiency of generation for the problem parameters close to the region of bistability. Spontaneous switching between the branches can explain the existence of high effective and high intensive generation that occurs at the special correlation between values of second dispersion coefficients and pulse duration. Crossing the boundary between regions 4 and 1 results in splitting (merging) of the solution, so conditions for mutual compensation of self-action and phase mismatching in order to achieve greater efficiency can be get.

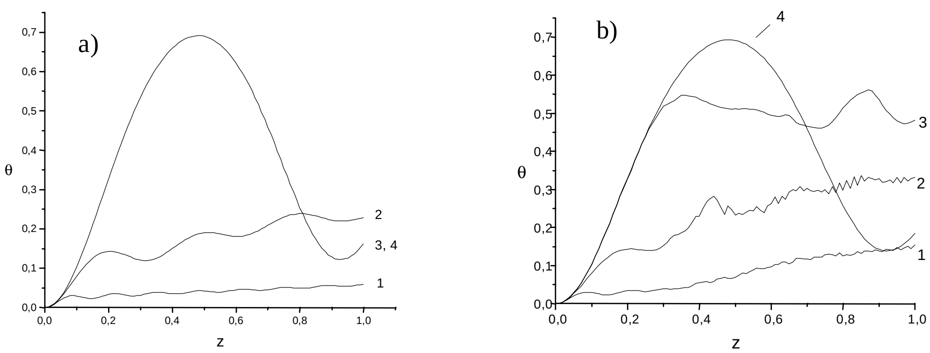


Fig.2 Evolution of SHG efficiency θ for various values of β :1 (1),0.5 (2), 0 (3), and $\alpha = 0$ (4) for dispersion values $D_1 = D_2 = 0.0000625$ (a) and $D_1 = D_2 = 1$ (b). Other parameters values: $\alpha = 25$, $\gamma = 4$, $\nu = 0$, $\Delta n = -2$, $\tau = 4$

4. RESULTS OF MODELLING

4.1 Various lifetime of cubic gratings.

Possibility of the full or partial compensation of self-action by the appropriate chose of nonlinear medium with different lifetime of cubic gratings (parameter β) was shown above in the frame work of long pulse approximation. Numerical simulation of the problem out of frames of long pulse approximation confirmed this possibility for the wide range of dispersion coefficients values under conditions of phase matching as well as phase mismatching. (Group velocity synchronism was supposed.)

Fig. 2 shows small values of conversion efficiency for $\beta = 1$ for both cases of small and large dispersion due to self-action. Self-action is negligible for $\beta = 0$ on the all length of propagation for small values of dispersions. For dispersions big enough self-action is compensated only in the vicinity of the input section. Though in both cases efficiency for $\beta = 0$ significantly exceeds the efficiency for $\beta = 1$. Length of SHG generation similar to the process without self-action is determined by dispersion. It should be also mentioned that phase mismatching increases divergence between dependencies under consideration, while phase matching increases the length of compensation.

4.2 Bistable regime SHG of femtosecond pulses.

Significant growth as well as significant reduction at the given distance can be achieved by simultaneous phase shifts of both waves at the appropriate section of the medium (Ref.7-9) for the parameters from the region of bistability or close to this region. This occurs due to the switching to the higher or lower effective branch of generation. Simultaneous phase shifts can be realized in practice by two different ways. First, one can insert a transparent plate into the medium at the appropriate section. Second, phase shifts can be achieved by the interaction of propagating pulses with the additional laser pulse that is applied in cross direction at the sections of phase shifts. Due to nonlinear self-action this pulse can be created required phase shift.

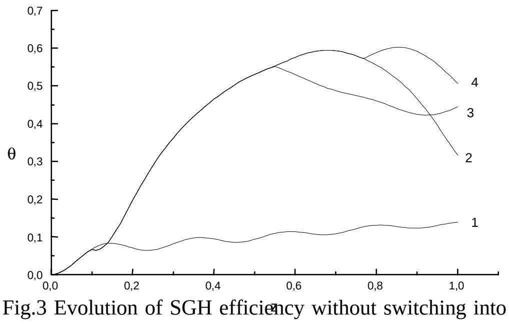


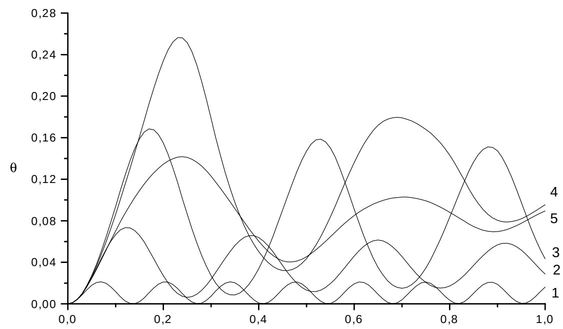
Fig.3 Evolution of SGH efficiency without switching into high-effective regime (1) and for switching $\Delta\varphi$ = -3 (2) ; $\Delta\varphi$ =-1.5 (3); $\Delta\varphi$ =-2.5 (4) at the corresponding sections. Parameters values: α =16, γ =4, Δz =-2.0, $A_1 = A_2 = 0.001$, $\nu = 0$, τ =10

It should be stressed that for the correspondingly small difference between quadratic and cubic non-linearity ($\alpha \sim 2\gamma$) switching can only keep high level of generation without its remarkable increase, which is true for both long and short pulses, the last being affected significantly by the second order dispersion. Big values of cubic non-linearity in the case of SHG of femtosecond pulses are more interesting for practice. In this case significant growth of generation level can be achieved by the first switching. The further switchings helps to maintain achieved level of generation (fig.3). In the case shown in fig.3 efficiency growth is up to 6 times and achieves 60%.

4.3 Effective generation far from phase synchronism.

As is well known phase mismatching ($\Delta z \neq 0$) results in the significant reduction of generation or even its suspension in the case of pulses in picosecond diapason when self-action can be neglected ($\alpha << \gamma$). The same behavior takes place for the strong self-action (for $\alpha > \gamma$) under conditions of phase synchronism. At the same time it is possible to get effective SHG generation under conditions of strong self-action and big phase mismatching (Ref.10)

Fig.4 demonstrates the possibility of getting high efficiency under conditions of big phase mismatching. Efficiency values in the absence of self-action do not exceed 2% for the considering great (Δz =-45) phase mismatching. Increase of the initial intensity results in highgrowth of self-action and significant (more than 10 times) increase of SHG efficiency. Maximal possible efficiency is determined by the optimal correlation between self-action and mismatching (for the considering mismatching $\alpha = 35$). Further increase of the basic wave initial intensity only reduces efficiency. The reverse statement is true: for all ratios of parameters α and γ, such that $\alpha > \gamma$, there is the optimal value of the coefficient Δz of phase mismatching and this value is negative.



Pic.4 Evolution of SGH efficiency far from phase synchronism (Δz =-45) for the following values of self-action parameter α : 0 (1), 15 (2), 25 (3), 35 (4), 40 (5). Other parameters values: γ =4, $\nu = 0$, $A_1 = A_2 = 0.0000625$, τ =4, $\beta = 1$.

Using the developed technology of efficiency increase by phase shifts, the process of generation can be essentially improved. Switching to the higher effective branch of generation in the domain of the first efficiency maximum provides more effective generation with the efficiency that varies insignificantly along the medium. All the results are true for the greater influence of the second order dispersion. It should be mentioned that in this case essential growth of efficiency occurs as well along the big part of the medium accompanied by oscillation on the background of the growth.

4.4 Effective SHG far away from group velocity matching.

SHG efficiency can be increased due to the partial suppression of self-action when the system is far away from group velocity matching, though this growth is not big enough (several percents for phase mismatching to two times for phase matching at $\nu = 10$) (fig.5). We demonstrate in this case the possibility to get high effective generation using the method of switching. Numerical simulation showed decrease of efficiency at the group velocity mismatching growth, though even for so big group velocity mismatching as $\nu = 10$ maximum values of efficiency are about 40%. Insertion of several phase shifts stabilizes SHG efficiency on the achieved level. Big values of dispersion (about 1) make the situation worse especially for group velocities far from synchronism, but even in this case a number of switching improve situation. We think that small group velocity mismatching together with self-action and phase mismatching can compensate influence of each other. This statement is due to the analogy of the process under consideration with the wave front reversal due to four-wave following interaction (Ref.11,12).

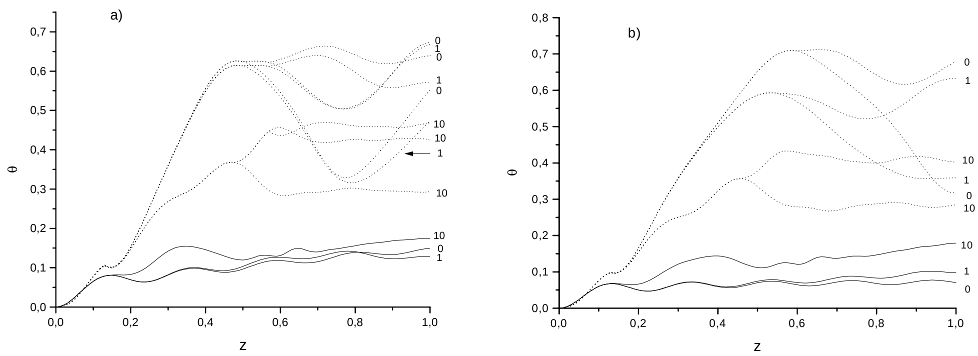


Fig.5. Evolution of SHG without (solid lines) insertion of phase shifts (solid lines) and with a number of phase shifts (dotted lines) for phase mismatching $\Delta k = -2$ (a) and phase matching (b). Numbers on the plot indicate values of parameter ν . Other values $\alpha = 1\sim$, $\gamma = 3$, $\beta = 1$, $\tau = 3$, $D_1 = D_2 = 0.00001$. Sections of switching correspond to the points of curves splitting.

5. CONCLUSIONS

We have shown here that the main factor that restrains high effective SHG generation – self-action –can be overcome outside the frames of phase synchronism using the method of inserted phase shifts. A number of switching to the higher effective branch of generation can strongly increase efficiency of generation and provide its maintenance on the achieved level. The method is available for the various cases of generation, including generation under phase and/or group velocity matching/mismatching and sufficient big dispersion. Thus the new class of optical bistable system can be created with high values of efficiency and switching velocity on the basis of insertion of additional phase shift.

We show also other ways to increase efficiency of generation - by choosing the medium with different lifetime of cubic gratings or with the special ratio of self-action and phase mismatching. In the first case the process is similar to the absence of self-action. Slight group velocity mismatching promotes generation as well.

Strong enough phase mismatching can rise efficiency up to several times in comparison with the case of phase synchronism. These results can be used for cascade generation of harmonics with the first SHG occurring with high efficiency (more than 70%) close to phase synchronism and the next doubling of high intensive pulse – far from phase synchronism with the efficiency of about 35%. It is possible to realize the further growth of generation using a phase shifts at the appropriate sections of the medium.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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